ISI – Bangalore Center – B Math - Physics I – Mid Semestral Exam

Date: 8th September 2015. Duration of Exam: 3 hours

Total marks: 35 ANSWER ALL QUESTIONS

Q 1. [Total Marks: 1+1+1+2+2=7]

FOR Q1 only the final answers need to be given; Calculation to get the final answer can be done in the rough pages but need not be shown.

1a.) True or False? :At the end of each cycle $\Delta W = \Delta Q$, for BOTH reversible AND irreversible engines (the symbols have the usual meaning)

1.b) True or False? : When a system undergoes isothermal transformation and does not do any external work in these transformations, the maximum amount of work that can be extracted is related to change in Helmholtz free energy.

1.c) True or False: When sugar dissolves in a cup of water at standard room temp and pressure, $\Delta S_{water+sugar} > 0$ and $\Delta G_{water+sugar} < 0$

1d.) When heat is conducted at a steady rate from a reservoir at a higher temp to a reservoir at a lower temp by a conducting rod with a steady temperature profile, which ones are FALSE statements?

i)entropy of each reservoir increases while there is no change entropy of the conducting rod remains the same.

ii) entropies of each reservoir and the rod increases

iii) entropy of the hotter reservoir decreases and entropy of the rod remains the same.

Iv) entropy of the reservoir at lower temp increases and entropy of the rod increases.

1.e) On a TS diagram, the slope of a line representing isobaric transformation is given by a function of C_v and T? What is that function. [Just the function required, not its derivation]

Q2 . [Total Marks: 5]

Suppose dQ is the infinitesimal heat necessary to raise the temperature of a unit mass of a substance by temperature dT while one of the state quantity x is kept constant. Show that the corresponding specific heat C_x is related to variation of the internal energy U with respect to volume V and temperature T in the following way:

$$C_{x} = \left(\frac{dQ}{dT}\right)_{x} = \left(\frac{\partial U}{\partial T}\right)_{V} + \left\{\left(\frac{\partial U}{\partial V}\right)_{T} + P\right\}\left(\frac{\partial V}{\partial T}\right)_{x}$$

Q3. [Total Marks: 5]

An ideal gas of one mole with volume V is separated from a vacuum chamber of volume2V by a wall which develops a slow leak and over time the gas fills up the entire volume 3V. Assume that the whole system is insulated adiabatically.

3a.) Which of the following statements is true, "The initial and final states lie on an adiabatic curve" OR the statement "The initial and final states lie on an isothermal curve"?

3b.) What is the change of entropy of the universe in this case? ONLY FINAL EXPRESSION, derivation can be in rough pages but need not be shown.

Q4. [Total Marks: 5]

Assume that two distinct adiabatic curves on a PV diagram intersect at a point A. Let an isothermal curve intersect both these adiabatic curves at points B and C.

4a.) Draw this on a PV diagram, and show that an engine running between A, B and C will violate the second law of thermodynamics.

4b.) What conclusion can one draw from part a.) above?

Q5. [Total Marks: 7]

5a.) Is the change in entropy in a system when it undergoes a quasi static adiabatic transformation positive, negative or zero?

5b.) Show that the quantity $V\left(\frac{\partial T}{\partial V}\right)_{S}$ which represents the change in temperature in quasi static

adiabatic volume expansion of a gas is related to C_V and parameters related to equation of state in the following way:

$$V\left(\frac{\partial T}{\partial V}\right)_{S} = -\frac{VT}{C_{V}}\left(\frac{\partial P}{\partial T}\right)_{V}$$

5c.) Use this to derive the well known result $T \propto V^{\frac{-R}{C_v}}$ for ideal gas.

5d.) What can we conclude about temperature variation of quasi static adiabatic expansion of any gas?

Q6. [Total Marks: 6]

6a.) Derive the equation
$$\left(\frac{\partial C_P}{\partial P}\right)_T = -T \left(\frac{\partial^2 V}{\partial T^2}\right)_P$$

6b.) If the equation of state is given by PV = RT + B(T)P where B(T) is a function of temperature

only, then show that
$$C_P = -TP \frac{d^2B}{dB^2} + C_P (P=0)$$